NONSTEADY THERMAL BENDING OF MULTILAYERED PLATES WITH INTERNAL COOLING

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UDC 621.039:621.3.
038:535.313

We have derived approximate analytical expressions to estimate the nonsteady values of temperature and thermal bending for multilayered plates with internal cooling.

Great thermal stresses and strains may be generated under the action of high thermal loads in the heat-conducting walls of physicoenergetic installations. In order the reduce these strains and stresses, we make use of microchannel (porous) cooling systems which are brought as close as possible to the heated surface [1-6]. In such an event, the structure of the wall is reminiscent of a three-layered plate of thickness δ, including: 1) a thin heat-receiving layer of thickness δ_1; 2) a cooled layer of thickness h, with arbitrarily shaped microchannels (pores in communication) through which the coolant moves [2-6]; and 3) a comparatively thick base of thickness δ_2, through which is effected the removal from and supply to of the coolant to the cooled layer and the plate mountings. The total thickness of the heat-receiving wall and of the cooled layer (δ_1 + h), as a rule, does not exceed 0.1 δ. In the case of large-scale cooled plates, in order to reduce their weight, the base is lightened through utilization of a variety of honeycombed or foam structures [7-10].

Thermomechanical calculation of such nonuniform structures is extremely complex and calls for the use of considerable simplifications, even when using numerical methods to solve the equations of thermoelasticity. Estimates have been obtained in [11] for the nonsteady thermal bending of solid plates not subjected to cooling.

It is purpose of the present study to derive analytical expressions convenient for various estimates of the nonsteady values of thermal bending in multilayered plates with internal cooling, where the thermal load is applied from without.

In order to validate the approximate method of calculating displacements, let us estimate the characteristic time required for the heating of a cooled porous layer when a constant thermal load is applied. Owing to the thermal conductivity of the skeletal structure of the cooled layer, a portion of the heat absorbed by the first wall penetrates down to the base, the heating of the latter taking up a greater portion of the time than the heating of the thin first wall and of the cooled layer. It was demonstrated in [4] that the time required to establish a steady heat-exchange regime in the porous layer is determined by the characteristic times for the heating of the skeleton, i.e., $\tau_s = C_s/\alpha_V$ and $\tau_\ell = C_\ell/\alpha_V$ for the liquid. Since the optimum porosity of the channel (porous) cooling systems is close to 50% [4] and the quantity α_V in the case of water cooling, as a rule, exceeds 10^7 W/(m^3·K), then for the majority of materials and structures forming the porous layer the times $\tau_\ell \approx \tau_s < 0.2$ sec, i.e., are rather small. If the duration of the thermal effect is small in comparison with τ_s, porous cooling is useless [4]. However, if the duration of the thermal effect is significantly greater than the time τ_s, the deformation of the plate depends significantly on the nonsteady temperature profile in the base, into which the heat has penetrated as a consequence of the conduction of heat from the skeleton of the pooled layer. It is precisely this case that we examined below.

According to [5], the influence exerted by a porous cooled layer functioning as a heat sink on the heat-receiving wall and base can be characterized by the heat-transfer coefficient's α_1 of the first wall and α_0 for the base, these being related to the volumetric transfer of heat α_V from the uniform porous layer by the following expressions:

Moscow Engineering Physics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 6, pp. 1005-1010, December, 1989. Original article submitted June 15, 1988.

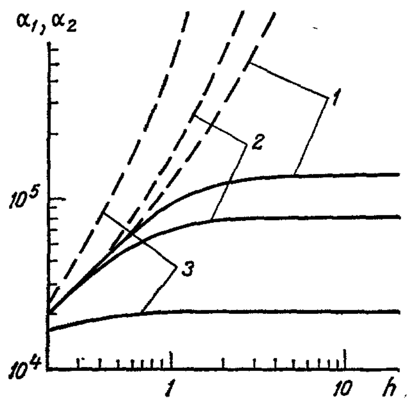

Fig. 1. The effect of the thickness of the cooled porous layer of copper (1), molybdenum (2), and invar (3) on the effective release of heat from the first wall (solid lines) and bases (dashed lines) when α_V = 10^8 W/(m^3·K), $\lambda_s = \lambda_{met}/3$. h, mm; α_1, α_2 W/(m^2·K)

$$\alpha_1 = \sqrt{\lambda_s \alpha_v}\,\mathrm{th}\left(h\sqrt{\frac{\alpha_v}{\lambda_s}}\right), \quad \alpha_0 = \sqrt{\lambda_s \alpha_v}\,\mathrm{sh}\left(h\sqrt{\frac{\alpha_v}{\lambda_s}}\right). \tag{1}$$

It follows from (1) that when α_V = const, with an increase in the thickness of the cooled layer, the transfer of heat from the first wall reaching the limit $\sqrt{\lambda_s \alpha_v}$, while the heat transfered from the base increases exponentially (Fig. 1). The quantity α_0 is convenient from the standpoint that it allows us for a given external uniform heat load q immediately to estimate the steady-state temperature of the base: $T_0 = T_\ell + q/\alpha_0$. For more complex structures, the average quantity α_0 must necessarily be determined by experiment.

Thus, with heating durations of $t \gg \tau_s$ and τ_ℓ the temperature of the base to which the cooled porous layer is attached and that of the heat-receiving wall are virtually coincident with the temperature state of a uniform solid plate whose surface z = 0 is heated by a flow of heat with density q and is simultaneously cooled by means of a cooling heat carrier with a heat-transfer coefficient of α_0. The nonsteady distribution of temperature in such a plate, with uniform heating by a flow of heat with density q, W/m^2, is described by the following heat-conduction equation:

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{2}$$

with the boundary conditions

$$T = T_\ell = \mathrm{const} \quad \text{when} \quad t < 0; \tag{3}$$

$$-\lambda \frac{\partial T}{\partial z} = q - \alpha_0 (T - T_\ell) \quad \text{when} \quad z = 0, \; t \geqslant 0; \tag{4}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=\delta_2} = 0. \tag{5}$$

The solution of this boundary-value problem for q = const has the form

$$T(z, t) = T_\ell + \frac{q}{\alpha_0} - \sum_{n=1}^{\infty} C_n \left(\sin\beta_n \frac{z}{\delta_2}\,\mathrm{tg}\,\beta_n + \cos\beta_n \frac{z}{\delta_2}\right)\exp(-\beta_n^2\,\mathrm{Fo}). \tag{6}$$

Here β_n represents the roots of the equation $\beta_n \tan\beta_n$ = Bi; Bi = $\alpha_0\delta_2/\lambda$;

$$C_n = \frac{2\mathrm{Bi}}{\beta_n^2 + \mathrm{Bi}^2 + 2\mathrm{Bi}\sin^2\beta_n + (\beta_n^2 - \mathrm{Bi}^2)\sin 2\beta_n/2\beta_n}. \tag{7}$$

The derived solution found in (6) exhibits the following singularities. If the thermal load is constant and acts for an unlimited length of time, the plate will pass, over time, from one isothermal state ($T = T_\ell$ when t = 0) to another ($T = T_\ell + q/\alpha_0$ as $t \to \infty$). The temperature of the face surface z = 0 of the plate is stabilized most quickly. The characteristic time within which the surface temperature reaches the asymptotic level $T_\ell + q/\alpha_0$ is $\tau_p \sim \lambda^2/a\,\alpha_0{}^2$, which corresponds to the Fourier number Fo $\approx 1/\mathrm{Bi}^2$. Hence, it follows that to the extent that the Biot number exceeds 1, the earlier the plate surface temperature becomes stabilized. Thus, with λ = 10 W/(m·K), $a > 10^{-5}$ m^2/sec, $\alpha_0 > 10^4$ W/(m^2·K), which is commensurate with the time required to heat the cooled layer.

When subjected to the action of a nonsteady transverse temperature profile such as that presented in (6), the free plate will bend in the direction of the heat flow. The only possibility of analytically evaluating the thermal bending of this plate is offered by the theory of thin plates [3, 4]:

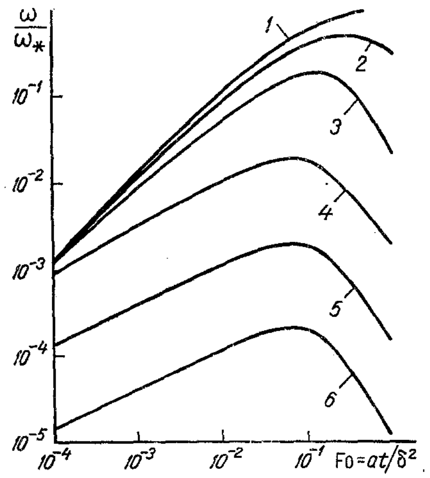

Fig. 2

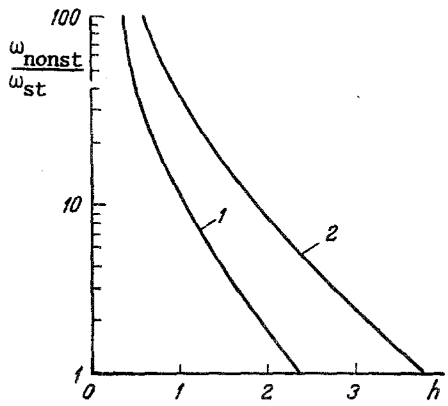

Fig. 3

Fig. 2. Variation in plate bending over time as a function of the Biot number on application of a constant thermal load from the pooled side: 1) Bi = 0; 2) 1; 3) 10; 4) 10^2; 5) 10^3; 6) 10^4.

Fig. 3. The effect of the thickness of the pooled porous layer on the ratio of maximum bending deformation (10) to steady deformation [3, 4] of a pooled three-layered plate of molybdenum when δ_1 = 1 mm, δ = 200 mm, λ_s = 43 W/(m × K), $\alpha_V = 10^8$ W/(m³·K) (1) and $3 \cdot 10^7$ W/(m³·K) (2).

TABLE 1. Value of the Complex $\beta\sqrt{a}$ for a Number of Materials, Based on the Data from [4]

Material	$\beta\sqrt{a}$, 10^{-6} m/K $\sqrt{\text{sec}}$	Material	$\beta\sqrt{a}$, 10^{-6} m/K $\sqrt{\text{sec}}$
Copper	18.1	Silicon	2.4
Beryllium	8.2	Silicon carbide	1.1-3.1
Molybdenum	3.6	NKD Invar 32	0.09
Tungsten	3.4		

$$\omega_{bend} = \omega_0 \left(\frac{r}{b}\right)^2, \quad \omega_0 = \frac{6b^2\beta}{\delta_2^3} \int_0^{\delta_2} T(z) \left(\frac{\delta_2}{2} - z\right) dz, \tag{8}$$

where ω_0 represents the direction of plate bending (the difference between the axial displacements at the center from those at the edge of the plate). If we substitute (6) into (8), we obtain the change in the direction of the bend over time:

$$\frac{\omega_0(t)}{\omega_*} = \sum_{n=1}^{\infty} \frac{C_n}{\beta_n^2} \left(\frac{1}{\text{Bi}} + \frac{1}{2} - \frac{1}{\text{Bi} \cos \beta_n}\right) \exp(-\beta_n^2 \text{Fo}). \tag{9}$$

Here $\omega_* = \beta q b^2/4\lambda$ represents the deflection of the uncooled plate in the quasisteady regime [11], in which a parabolic temperature profile is established through the thickness of the plate. The results from calculations carried out in accordance with formula (9) are shown in Fig. 2. With Bi = 0 (the case of an uncooled plate has been analyzed in detail in [11]) the bending increases as heating proceeds, initially as $\omega_0 \approx \omega_* 12\text{Fo}$, and then with Fo ≥ 0.3 the bending is stabilized at a level of ω_*.

An exact solution was initially derived in [12] for the steady thermal deformations of the end face on a cylinder of finite dimensions. It follows from this solution that with unilateral heating and cooling of the cylinder, where the opposite face is insulated against heat, the strains generated within the face subjected to simultaneous and cooling, as a minimum, exceeds by a factor of two the strains calculated in the approximation of the theory of thin plates. Therefore, the bending in expressions (8) and (10) and in (9) for ω_* must be increased by a factor of 2.

With large Biot numbers (Bi ≥ 100), characteristic of large-scale plates, the bend is initially increased, and then reduced as a consequence of the equalization of the temperature profile in the base. The time required to attain the maximum bending is independent of heating and cooling intensity and amounts to Fo ≈ 0.1 (more precisely, $1/4\pi$), while the magnitude of the maximum bending is inversely proportional to the thickness and release of heat from the plate:

$$\omega_{\max} \approx \frac{3\beta b^2 q}{2\pi\delta\alpha_0} = \frac{3}{2\pi^2}\,\frac{\beta Q}{\delta\alpha_0}. \tag{10}$$

Here $Q = q\pi b^2$ represents the total thermal load of the plate. Thus, for typical parameters $b = 0.5$ m, $\delta = 0.2$ m, $\beta = 10^{-5}$ K^{-1}, $\lambda = 10$ W/(m·K), $\alpha_0 = 10^5$ W/(m^2·K) (i.e., Bi = 2000), $a = 3\cdot10^{-5}$ m^2/sec the time required to attain maximum bending amounts to 100 sec, while the thermal load which produces a bend of 1 μm is 16 kW/m^2.

With Bi ≥ 100 and Fo ≤ 0.2 the change in the deflection of the cooled plate is approximated by the expression

$$\omega_0(t) = \omega_{\max}\, 4\pi\left(\sqrt{\frac{\mathrm{Fo}}{\pi}} - \mathrm{Fo}\right). \tag{11}$$

Hence it follows that for given heat load, effective heat-load time (Fo < 0.1) and plate cooling intensity, the minimum bending will be found in that plate fabricated out of material with a minimum magnitude for the complex $\beta\sqrt{a}$. Among the materials listed in Table 1, the poorest from this standpoint is copper, while the best is Invar, distinguishing itself by its low thermal expansion and thermal diffusivity.

Thus, the bending of a plate cooled from the heating side initially increases, reaching a maximum, and then it diminishes to zero. The base of a multilayer plate behaves in similar fashion, given that it is elastically linked to a thin cooled layer and a heat-receiving wall, the only difference being that the bending of the entire plate does not reduce down to zero, but to some steady value defined by the temperature profile in the first wall and by the cooled layer, and by the rigidity of these last two factors. If we assume the rigidity of a multilayered plate to be equal to that of the base, it is not difficult to evaluate the magnitude of the steady bending from the maximum, as was done in [3, 4]. It follows from Fig. 3 that the maximum of the nonsteady bending of the base, i.e., Eq. (10), may significantly exceed the bending of the entire plate in the steady regime [3, 4]. Intensification of heat exchange (an increase in α_V) in the cooled layer, just as an increase in the thickness of the layer (with a constant α_V), leads to a reduction in the magnitude of the nonsteady bending of the base in a multilayered plate with internal cooling.

Using expression (10), (11), (1), as well as the recommendations dealing with the calculation of α_V and λ_S from [4], we can study the influence exerted by the rate of coolant flow, the properties of the skeletal structure, the liquids and the bases, the geometric parameters and the structures of the cooled layer and the base on the extent to which cooled plates are subjected to bending under nonsteady conditions.

NOTATION

λ_S, thermal conductivity of a porous skeletal structure for the cooled layer; C_S and C_ℓ, heat capacities of the skeletal structure and of the liquid in calculation for a unit volume of porous medium; α_V, volumetric coefficient of heat transfer in the porous skeletal structure of the liquid; β, coefficient of thermal expansion.

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AN APPROXIMATE ANALYTICAL SOLUTION FOR A THREE-DIMENSIONAL HEAT-CONDUCTION PROBLEM IN AN AIR-RADIATION HEATING SYSTEM

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UDC 536.2:536.68

We offer a method for the calculation of the heat transferred from a system, this method being based on replacement of the three-dimensional process by a combination of a two-dimensional and a one-dimensional process in various cross-sectional planes of the heater channels.

The need has recently arisen to conduct thermotechnical calculations related to a variety of design solutions for air-radiation heating systems, governed by the transfer of heat from the surfaces of barriers through whose thicknesses regular channels have been cut, and these are heated by means of circulating hot air (Fig. 1). Rigorous formulation of the steady-state problem of calculating the influx of heat from such a system reduces to the description of the three-dimensional process determined by the Poisson equation, whose precise analytical solution can not be obtained.

We will look for the solution of the formulated problem by taking into consideration the following assumptions: the replacement of the three-dimensional process by a combination of a two-dimensional process within the plane of the lateral cross section of the channels and of the one-dimensional process in the longitudinal cross section of these planes will introduce no significant errors; the temperatures τ_c and t can be assumed to be constant for each lateral cross section of the channel, while the quantities t_1, t_2, α_0, α_1, α_2, λ_0, λ_a, c_a, η_a can be assumed to be constant within the framework of the entire system; we need not take into consideration the heat released from the ends of the barrier, nor need we make provision for the relationship between the amount of heat transferred out of the channel and the location of the latter.

We are familiar with at least three means of solving the two-dimensional heat-conduction problem in the plane of the lateral cross section of regular linear heating elements. Ananikyan's and Pavlov's [1] use of the method of sources and sinks offers no rigorous physical basis and is not applicable to the case $t_1 \neq t_2$.

The solution of the Schwartz–Christoffel integrals (the conformal transformation method) obtained by Sander [2] for the problem in the plane of the lateral cross section of the channels, because of its complexity, leads to resulting differential equations in the longitudinal cross-sectional plane that are insoluble in quadratures.

Most appropriate to the solution of the formulated problem is the Faxen–Rydberg–Huber method. In [3] Faxen published a solution for the two-dimensional heat-conduction problem related to a uniform panel with regular linear heating elements for the case $t_1 = t_2$:

$$\frac{vb}{\pi A} = \frac{k_2 - k_1}{k_0} y - |y| + 2\frac{\lambda_0}{k_0} + \frac{b}{\pi}\sum_{i=1}^{\infty}\frac{\cos(2\pi ix/b)}{i} \times \quad (1)$$

Central Scientific-Research Institute of Industrial Construction, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 6, pp. 1010-1015, December, 1989. Original article submitted April 26, 1988.